

## Chp 10 Review

(17)  $x = \ln t$      $y = 1 + t^2$      $t=1$

$$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 2t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{\frac{1}{t}} = 2t^2 \quad \boxed{\frac{dy}{dt}|_{t=1} = 2} \quad \checkmark$$

(19)  $r = \theta$      $\theta = \pi/4$      $\frac{dr}{d\theta} = 1$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}(\sin\theta) + (\cos\theta)r}{\frac{dr}{d\theta}(\cos\theta) - (\sin\theta)r} = \frac{1(\sin\theta) + (\cos\theta)(\theta)}{1(\cos\theta) - (\sin\theta)(\theta)}$$

$$\frac{dy}{dx} = \frac{\sin \pi/4 + (\cos \pi/4) \pi/4}{\cos \pi/4 - (\sin \pi/4) \pi/4} = \frac{8\left(\frac{\sqrt{2}}{2} + \frac{\pi\sqrt{2}}{8}\right)}{8\left(\frac{\sqrt{2}}{2} - \frac{\pi\sqrt{2}}{8}\right)} =$$

$$\boxed{\frac{dy}{dx} = \frac{4\sqrt{2} + \pi\sqrt{2}}{4\sqrt{2} - \pi\sqrt{2}}} = \frac{\sqrt{2}(4+\pi)}{\sqrt{2}(4-\pi)} = \boxed{\frac{4+\pi}{4-\pi}} \quad \checkmark$$

(21)  $x = t \cos t$      $y = t \sin t$   
 $x'(t) = \cos t - t \sin t$      $y'(t) = \sin t + \cos t \cdot t$

$$\frac{dy}{dx} = \frac{\sin t + t \cos t}{\cos t - t \sin t} \quad \checkmark \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \frac{(cost + cost - tsint)(cost - tsint) + (-sint - sint + tcost)(sint + tcost)}{(cost - tsint)^2}$$

$$\cancel{2cost} \rightarrow 2\cos^2 t - t \cos t \sin t - \cancel{2t \cos t \sin t} + t^2 \sin^2 t + 2\sin^2 t + 2t \sin t \cos t + t \cos t \sin t + t^2 \cos^2 t$$

$$\frac{d^2y}{dx^2} = \frac{2(\cos^2 t + \sin^2 t) + t^2(\sin^2 t + \cos^2 t)}{(cost - tsint)^3} = \boxed{\frac{d^2y}{dx^2} = \frac{2+t^2}{(cost - tsint)^3}} \quad \checkmark$$

(23) lowest point = Minimum y value

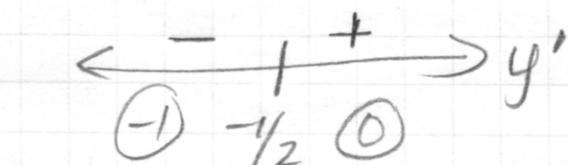
$$y(t) = t^2 + t + 1$$

$$y'(t) = 2t + 1$$

$$0 = 2t + 1$$

$$t = -\frac{1}{2}$$

$$x(t) = t^3 - 3t$$

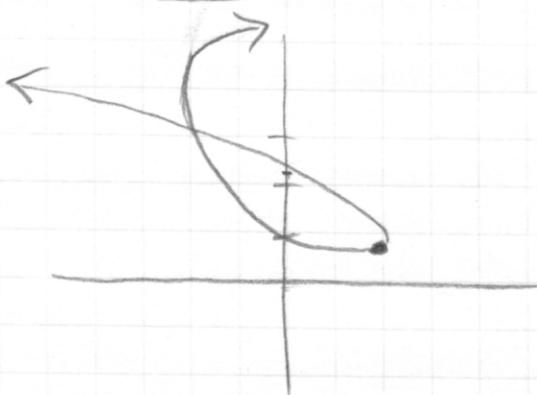


$$x(-\frac{1}{2}) = -\frac{1}{8} + \frac{3}{2} = \frac{-1+12}{8} = \frac{11}{8}$$

$$x(-\frac{1}{2}) = \frac{11}{8} \quad y(-\frac{1}{2}) = \frac{3}{4}$$

$$\boxed{(\frac{11}{8}, \frac{3}{4})}$$

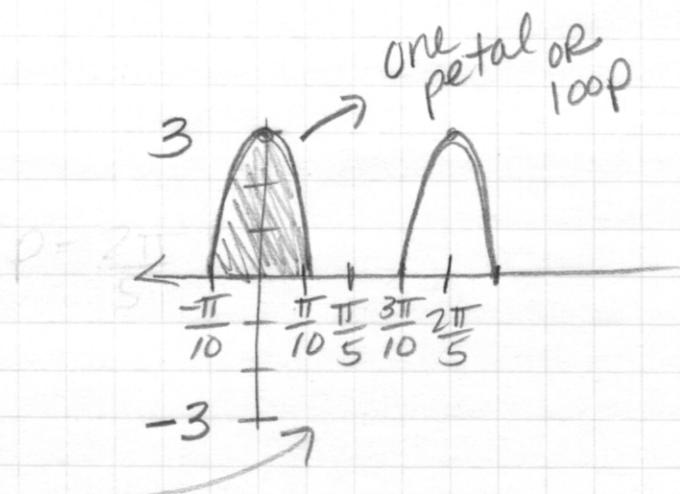
$$y(-\frac{1}{2}) = \frac{1}{4} - \frac{1}{2} + 1 \\ \frac{1}{4} - \frac{2}{4} + \frac{4}{4}$$



(27)  $r^2 = 9 \cos 5\theta$

$$r = \pm 3\sqrt{\cos 5\theta}$$

$$P = \frac{2\pi}{5} \quad \text{10 petals Total}$$



$$10 \left[ \frac{1}{2} \int_{-\pi/10}^{\pi/10} (9 \cos 5\theta) d\theta \right] = 5 \int_{-\pi/10}^{\pi/10} 9 \cos 5\theta d\theta =$$

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(29)  $r=2$  &  $r=4 \cos \theta$

$$2 = 4 \cos \theta$$

$$\frac{1}{2} = \cos \theta$$

$$\text{Polar} \rightarrow (2, \pm \frac{\pi}{3})$$

$$\theta = \pi/3 \text{ and } 5\pi/3$$

$$\begin{cases} (1, \sqrt{3}) \\ (1, -\sqrt{3}) \end{cases} \rightarrow \text{Rectangular}$$

$$x = r \cos \theta$$

$$x = 2 \cdot \cos \pi/3 = 1 \quad (1)$$

$$y = r \sin \theta$$

$$y = 2 \cdot \sin \pi/3 = \sqrt{3} \quad (\sqrt{3})$$

$$x = 2 \cos 5\pi/3 = 1 \quad (1)$$

$$y = 2 \sin 5\pi/3 = -\sqrt{3}$$

$$(31) \quad r = 2\sin\theta$$

$$r = \sin\theta + \cos\theta$$

$$2\sin\theta = \sin\theta + \cos\theta$$

$$\sin\theta = \cos\theta$$

$$\theta = \pi/4 \text{ & } 5\pi/4$$

$$\frac{1}{2} \int_0^{\pi/4} (2\sin\theta)^2 d\theta + \frac{1}{2} \int_{\pi/4}^{3\pi/4} (\sin\theta + \cos\theta)^2 d\theta$$

$$1.071 \checkmark$$

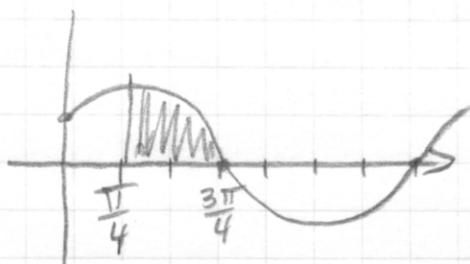
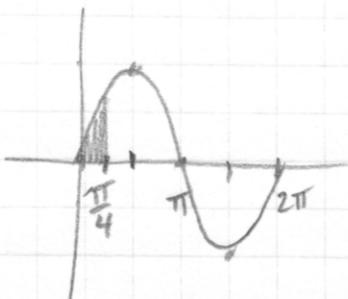
$$\sin\theta + \cos\theta = 0$$

$$\sin\theta = -\cos\theta$$

$$\theta = 3\pi/4 \text{ & } 7\pi/4$$

$$r = 2\sin\theta$$

$$r = \sin\theta + \cos\theta$$



$$(33) \quad x = \frac{3t^2}{2} \quad y = 2t^3 \quad 0 \leq t \leq 2$$

$$AL = \int_0^2 \sqrt{(6t)^2 + (6t^2)^2} dt$$

$$\begin{aligned} u &= 1+t^2 \\ du &= 2t dt \\ \frac{du}{dt} &= 2t \end{aligned}$$

$$AL = \int_0^2 \sqrt{36t^2 + 36t^4} dt = \int_0^2 6t \sqrt{1+t^2} dt$$

$$\int_1^5 6t \sqrt{u} \frac{du}{2t} = 3 \int_1^5 u^{1/2} du = 3 \left[ \frac{2}{3} u^{3/2} \right]_1^5 =$$

$$2 \cdot 5\sqrt{5} - 2(1) = 10\sqrt{5} - 2 \checkmark$$

$$(35) \quad r = \frac{1}{\theta} \quad \frac{dr}{d\theta} = -\theta^{-2} = -\frac{1}{\theta^2} \quad \pi \leq \theta \leq 2\pi$$

$$AL = \int_{\pi}^{2\pi} \sqrt{\left(\frac{1}{\theta}\right)^2 + \left(-\frac{1}{\theta^2}\right)^2} d\theta =$$

$$\int_{\pi}^{2\pi} \sqrt{\frac{1}{\theta^2} + \frac{1}{\theta^4}} d\theta = \int_{\pi}^{2\pi} \sqrt{\frac{\theta^2 + 1}{\theta^4}} d\theta =$$

$$\int_{\pi}^{2\pi} \frac{1}{\theta^2} \sqrt{\theta^2 + 1} d\theta \quad * \text{Trig Sub!} \quad * \text{use } \underline{\text{Calc}} = 0.712$$

$$(37) \quad x = 4\sqrt{t} \quad y = \frac{1}{3}t^3 + \frac{1}{2}t^{-2} \quad 1 \leq t \leq 4$$

$$\frac{dx}{dt} = 2t^{-1/2} \quad \frac{dy}{dt} = t^2 + -1t^{-3}$$

$$SA = 2\pi \int_1^4 \left( \frac{1}{3}t^3 + \frac{1}{2t^2} \right) \sqrt{(2t^{-1/2})^2 + (t^2 - \frac{1}{t^3})^2} dt$$

$$SA = 2\pi \int_1^4 \left( \frac{1}{3}t^3 + \frac{1}{2t^2} \right) \frac{(t^5+1)}{t^3} dt$$

$$SA = 2\pi \int_1^4 \left( \frac{1}{3}t^3 + \frac{1}{2t^5} \right) (t^5+1) dt$$

$$SA = 2\pi \int_1^4 \left( \frac{1}{3}t^5 + \frac{1}{3} + \frac{1}{2} + \frac{1}{2}t^{-5} \right) dt = 2\pi \left[ \frac{1}{18}t^6 + \frac{5}{6}t + \frac{1}{8}t^{-4} \right]$$

$$2\pi \left( \left[ \frac{4096}{18} + \frac{20}{6} - \frac{1}{2048} \right] - \left[ \frac{1}{18} + \frac{5}{6} - \frac{1}{8} \right] \right) = \frac{460.249\pi}{471,295\pi} \quad \boxed{0.98}$$